Control and Navigation of Formations of Car-Like Robots on a Receding Horizon

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***Abstract*— The formulation and solution of a minimum time optimal control problem for a formation conformed by nonholonomic car-like mobile robots and a virtual leader reaching a target zone in an environment that includes dynamic and static obstacles with arbitrary shapes, is provided in this paper. The proposed approach for solving the formation to target zone minimum time problem, is formulated using receding horizon control methodologies. Simulation results using the proposed methodology are also reported.**

I. Introduction

Over the last decades there has been a great deal of interest in the robotic and control research community for problems involving coordination and control of autonomous vehicles [7]. From the list of problems involving coordination and control of autonomous vehicle systems, the problem of formation control is probably one of a few that has attracted major attention due to a wide range of applications. In this paper we propose an approach for multiple car-like type robots driving in a desired formation to a desired goal region based on leader-following formation methods and receding horizon control methodologies. Such problem has great relevance to the scheduling and control of autonomous ploughs for snow shoveling of airports as being one of our target applications [10].

In the classical literature the formation driving methods are divided by the three main approaches: virtual structures [9], behavioral techniques [8], [5], and leader-follower methods [2], [11]. In this paper we focus of leader-follower methods.

Development of Receding Horizon Control, RHC ([6]) for applications of formation driving control is presently intensively studied by the controls research community. A system employing distributed receding horizon control for stabilization of robots in desired positions within the formation is presented in [1]. Other interesting works that address the formation control of multiple vehicles from the Receding Horizon Control perspective are presented in [4], and [3].

Our contribution in this paper is a general approach that accounts for car-like robots formation stabilization and a time optimal trajectory planning (enforced by a stability constraint) to reach a target region, also considering the response to dynamic changes in the environment. In our approach RHC is used for both, the leader trajectory planning to the desired goal area as well as the trajectory following of the formation in an specified pattern.

The paper is structured as follows: in Section II, necessary preliminaries are presented. Section III presents the problem formulation. In Section IV, the receding horizon approach is introduced. Section V summarizes the formation driving concept. In Section VI, optimization implementation details of the proposed scheme are presented and in Section VII a numerical example is provided. Final conclusions and remarks are presented in Section VIII.

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II. Preliminaries

A. Robots Configuration Space

Let $\psi_L(t) = \{x_L(t), y_L(t), \theta_L(t)\} \in \mathcal{C}$ denote the configuration of a virtual leader robot $R_L(\psi_L(t))$ and $\psi_i(t) = \{x_i(t), y_i(t), \theta_i(t)\} \in \mathcal{C}$, with $i \in \{1, \ldots, n_r\}$, denote the configuration for each of the n_r follower robots $R_i(\psi_i(t))$ at time t, where $\mathcal{C} = \mathbb{R}^2 \times \mathbb{S}^1 \times \mathbb{W}$ is the configuration space. The Cartesian coordinates $(x_i(t), y_i(t))$ for an arbitrary configuration $\psi_i(t) \in \mathcal{C}$, define the position of a robot $R_i(\psi_i(t))$ and $\theta_i(t)$ denotes its heading. In $\mathcal{C}$, $\mathbb{W}$ represents the set of all closed, bounded and connected environments for the robots in $\mathcal{F}$. The environment world $\mathbb{W}$ contains a finite number n_0 (maybe unknown) of compact obstacles collected in a set of regions $\mathcal{O}_{obs} \subset \mathbb{W}$ characterized by non self-intersecting polygons (static obstacles) and circles (radially bounded dynamic obstacles).

B. Kinematic Model and Constraints

The kinematics for any robots j, $R_j(\psi_j(t)) \in \mathcal{F}$, $j \in \{1, \ldots, n_r, L\}$, is described by the simple nonholonomic kinematic model:

$$\begin{aligned} \dot{x}_j(t) &= v_j(t)\cos\theta_j(t) \\ \dot{y}_j(t) &= v_j(t)\sin\theta_j(t) \\ \dot{\theta}_j(t) &= K_j(t)v_j(t), \end{aligned} \tag{1}$$

with $\bar{p}_j(t) = (x_j(t), y_j(t))$ as the position of the vehicle and $\theta_j(t)$ its heading at time t, defining the system states in $\psi_j(t) \in \mathcal{C}$, while the velocity $v_j(t)$ and curvature $K_j(t)$ are control inputs $\bar{u}_j(t) = (v_j(t), K_j(t)) \in \mathbb{R}^2$.

C. Model and Controller Parametrization

Let us define a time interval $[t_0, t_M]$ containing a finite sequence with M elements of nondecreasing times $\mathcal{T}(t_0, t_M) = \{t_0, t_1, t_2, \ldots, t_{k-1}, t_k, \ldots, t_{M-2}, t_{M-1}\}$, such that $t_0 < t_1 < \ldots < t_k < t_{M-1} < t_M$. Also, let us define a controller for a robot $R(\psi_j(t_0))$ starting from a configuration $\psi_j(t_0)$ by $\mathcal{U}_j(t_0, t_M, \mathcal{T}) := \{\bar{u}_j(t_0; t_1 - t_0), \bar{u}_j(t_1; t_2 - t_1), \ldots, \bar{u}_j(t_{M-1}; t_M - t_{M-1})\}$ which is characterized as a sequence of constant control actions. Each element $\bar{u}_j(t_k; t_{k+1} - t_k), k \in \{0, \ldots, M-1\}$, of the finite sequence $\mathcal{U}_j(t_0, t_M, \mathcal{T})$ will be held constant during the time interval $[t_k, t_{k+1})$ with length $t_{k+1} - t_k$, not necessarily uniform. In this spirit we notice how over a time interval $[t_0, t_M]$ a controller can be parametrized with a minimal amount of information contained in the sequences of constant control values $\mathcal{U}_j(\cdot)$ and switching times $\mathcal{T}(\cdot)$. In order to simplify our notation: 1) we will include the time t_M in $\mathcal{T}$, and 2) the relation between $\mathcal{U}_j(\cdot)$ and $\mathcal{T}(\cdot)$ becomes implicit such that $\mathcal{U}_j(t_0) \equiv \mathcal{U}_j(t_0, t_M, \mathcal{T})$ and $\mathcal{T}(t_0) \equiv \mathcal{T}(t_0, t_M)$.

By integrating the model in (1) over a given interval $[t_0, t_f]$, and holding constant control inputs from $\mathcal{U}_j(t_0)$ over each time interval $[t_k, t_{k+1})$, where t_{k+1}, $t_k \in \mathcal{T}(t_0)$, $k \in \{0, 1, \ldots, M-1\}$ (associated to each t_k we have an index k), we can derive the following model for the *transition points* at which control inputs change:

$$
\begin{aligned}
x_j(k+1) &= \begin{cases} x_j(k) + \frac{1}{K_j(k+1)} [\sin(\theta_j(k) + \\ K_j(k+1) v_j(k+1) \Delta t(k+1)) - \\ \sin(\theta_j(k))], \text{if} K_j(k+1) \neq 0; \\ x_j(k) + v_j(k+1) \cos(\theta_j(k)) \Delta t(k+1), \\ \text{if} K_j(k+1) = 0 \end{cases} \\
y_j(k+1) &= \begin{cases} y_j(k) - \frac{1}{K_j(k+1)} [\cos(\theta_j(k) + \\ K_j(k+1) v_j(k+1) \Delta t(k+1)) - \\ \cos(\theta_j(k))], \text{if} K_j(k+1) \neq 0; \\ y_j(k) + v_j(k+1) \sin(\theta_j(k)) \Delta t(k+1), \\ \text{if} K_j(k+1) = 0 \end{cases} \\
\theta_j(k+1) &= \theta_j(k) + K_j(k+1) v_j(k+1) \Delta t(k+1),
\end{aligned} \tag{2}
$$

where $x_j(k)$ and $y_j(k)$ are the rectangular coordinates and $\theta_j(k)$ is the heading angle for the configuration $\psi_j(k)$ at the transition point with index k, $v_j(k+1)$ and $K_j(k+1)$ are control inputs from $\mathcal{U}_j(k+1) := u_j(t_k; t_{k+1} - t_k)$ at time index $k+1$ in $\mathcal{T}(k+1) := t_{k+1}$, and $\Delta t(k+1) := (t_{k+1} - t_k)$ is the sampling time during the time interval $[t_k, t_{k+1})$, where $t_{k+1}, t_k \in \mathcal{T}(t_0)$. Observe from the previous notation that the indexing for the sequences $\mathcal{U}_j(\cdot)$ and $\mathcal{T}(\cdot)$ start at 1, such that $\mathcal{U}_j(1) = u_j(t_0; t_1 - t_0)$ and $\mathcal{T}(1) = t_1$. The previous model is a continuous time model in regard that for any $t \in [t_k, t_{k+1})$, $k \in \{0, 1, \ldots, M-1\}$, the configuration of $R_j(\psi_j(t))$ can be determined exactly given any initial conditions. On the other hand this model can also be seen as a discrete model evolving over the transition points of the control inputs.

The previous model and notation allows us to describe long trajectories exactly using a minimal amount of information such as: i) the initial configuration $\psi_j(t_0)$, ii) the sequence of switching times $\mathcal{T}$, and iii) the sequence of controls actions $\mathcal{U}_j$.

D. Controller Constraints

Constraints can be taken into account for each robot $R_j(\psi_j(k))$ limiting their control inputs by inequalities, $v_{min,j} \leq v_j(k) \leq v_{max,j}$ and $|K_j(k)| \leq K_{max,j}$, where $v_{max,j}$ is maximal forward velocity of the j-th vehicle, $v_{min,j}$ is limit of backward velocity and $K_{max,j}$ is the maximal control curvature. This values can be different for each robot $R_j(\psi_j(k)$. We will denote from now on the set of admissible controls for each follower $R_i(\cdot)$ and leader $R_L(\cdot)$ by $\mathbb{U}_i$ and $\mathbb{U}_L$, accordingly.

III. Problem Formulation

The minimum time optimal control for a formation $\mathcal{F}$ of n_r nonholonomic car-like mobile robots reaching a target zone is defined as our general goal. The proposed approach for solving the *formation to target zone minimum time problem*, is formulated using receding horizon control methodologies for both: i) the trajectory planning and controls of the virtual leader of the formation, and ii) the control of the follower formation where each individual robot performs trajectory tracking of a specific path generated by the traveled trajectory of virtual leader and a suitable change of coordinates for relative distances in curvilinear coordinates. To accomplish such a task, a target zone must be clearly defined by a higher control entity or planner. Such planner can be a planner as the one presented in [10].

Definition 1. (Target Region) A target region $S_F \subset \mathbb{W}$ is a convex compact region such that for any robot $R(\psi) \in S_F$, $\psi \in \mathcal{C}_{free}$.

For the proposed solution to the problem in question we will consider the following non restrictive assumption.

Assumption 1. (Desired Reachability) The virtual leader $R_L(\psi_L(t))$ can get from any initial configuration $\psi_L(t_0) \in \mathcal{C}_{free}$ at time t_0 to any other configuration $\psi_L(t_f)$, $t_f > t_0$, inside a defined compact ball $\mathbb{B}(\bar{p}_f, \epsilon)$ centered at the point $\bar{p}_f \in \text{Proj}_{\bar{p}}(\mathcal{C}_{free})$ with radius $\epsilon > r_{min} > 0$, such $\{\mathbb{B}(\bar{p}_f, \epsilon) \cap \text{Proj}_{\bar{p}}(\mathcal{C}_{obs})\} = \emptyset$, in some finite time $t_f - t_0$, where $\text{Proj}_{\bar{p}}(\cdot)$ is a projection operator on the Cartesian coordinates of a given set. The radius bound r_{min} is chosen based on some of the constraints associated with the maximal control curvature of the robots.

IV. Receding Time Horizon Approach

The main idea of receding horizon control is to solve a moving finite horizon optimal control problem for a system (represented by a dynamic model), starting from current states or configuration $\psi(t_0)$ over the time interval $[t_0, t_f]$, where δt is a sampling time, $0 < \delta t < t_f - t_0$, and under a set of constraints on the system states and control inputs. In this framework the length $t_f - t_0$ of the time interval $[t_0, t_f]$ is known as the control horizon. After a solution from the optimization problem is obtained on a control horizon, a portion of the computed control actions is applied on the interval $[t_0, \delta t + t_0]$, known as the receding step. This process is then repeated on the interval $[t_0 + \delta t, t_f + \delta t]$ as the finite horizon moves by *time steps* defined by the sampling time δt, yielding a state feedback control scheme strategy (assuming all states are available).

In the proposed approach for the motion planning and control of the formation leader, the horizon is divided in two: (1) a first segment which holds a constant sampling rate used for obtaining a refined immediate control input for the leader who generates desired trajectories for the followers, and (2) a second segment where the length of the time intervals between instances where control inputs change are also variables taking part on the planning problem of reaching the target region. Details on such construction are presented in Section VI. Regarding the receding horizon control of the followers, a standard approach (fixed sampling time and length horizon) for the trajectory tracking problem is taken [6].

A. Finite Time Horizon Optimal Control of Leader

Let us state the formulation of the receding finite time horizon control approach for the leader $R_L(\psi_L(t_0))$ to target zone S_F minimum time optimal control problem, denoted by $\mathcal{P}(t_0)$, starting from initial condition $\psi_L(t_0)$ at time t_0, as:

$$J(\psi_L(t_0),\mathcal{U}_L^{\circ}(t_0);T)^{\circ} = \min_{\mathcal{U}_L} \left\{ \int_{t_0}^{T} L(\psi_L(s),\mathcal{U}_L(s),s)ds \right\}. \tag{3}$$

In this formulation $\mathcal{U}_L(t_0)^{\circ} \in \mathbb{U}_L$ denotes the optimal controller that generates optimal predicted states $\psi_L^{\circ}(\cdot)$. The minimization of the predicted cost $J(\cdot)$ is subjected to a set of equality constraints $h(\cdot) = 0$ representing the system model in (2) over a finite time horizon of length T (initially chosen arbitrary large at t_0) and a set of inequality constraints $g(\cdot) \leq 0$ that impose system's states and control input constraints as well as artificially introduced constraints to guaranty stability properties (for more details see, [6]). The proposed stability constraint is of the form, $g_{S_F}(\psi_L(T)) \leq 0$, representing the condition that at some finite prediction time T, the last position state of the horizon with length T enters S_F.

The function to be minimized in $\mathcal{P}(\cdot)$, $L(\psi_L(\cdot),\mathcal{U}_L(\cdot),\cdot)$, penalize elapsed time for virtual leader until reaching S_F and local proximity of $\psi_L(\cdot)$ with unsafe regions in the environment (proximity penalties are only active in the vicinity of static or dynamic obstacles). $L(\psi_L(\cdot),\mathcal{U}_L(\cdot),\cdot)$ is chosen as a time-invariant, positive definite function, being a key element in developing stability properties for the leader-follower control scheme.

Assumption 2. Once position states of $\psi_L(\cdot)$ enter S_F at some time $\bar{t} \in [t_0, \infty)$, no additional cost will be acquired by the cost function.

B. Stability Result

With the previously introduced elements let us present a stability result for the leader robot receding horizon scheme, where due to the lack of space, a proof for this result its not provided.

Theorem 2. *Under Assumptions 1-2, given a target region S_f, and a feasible solution for $\mathcal{P}(t_0)$, the receding time horizon control scheme that iteratively solves the optimal control problem in (3) (including stability constraint $g_{S_F}(\psi_L(T)) \leq 0$), to obtain control inputs $\mathcal{U}_L(\cdot)^{\circ}$ for the Leader robot, asymptotically stabilizes and guides $R_L(\psi_l(\cdot))$ toward the target region S_F. Claim holds as long as perturbations on $J(\cdot)^{\circ}$ due to unexpected obstacles (acting on proximity penalties for $L(\cdot)$) denoted by $D(k,t_n)$, satisfy $D(k,t_n) < \int_{k\delta t_n}^{(k+1)\delta t_n} L(x(s),\mathcal{U}_L(s),s)ds$, between any two times $k\delta t_n$ and $(k+1)\delta t_n$, $k \in \mathbb{Z}^+$, where $\delta t_n := \Delta tn$.*

V. Formation Driving Concept

The formation driving method described in this section is based on a leader-follower approach, in which the followers $R_i(\psi_i(t))$, $i \in \{1, ..., n_r\}$ should follow a leader's trajectory. Such trajectory will be divided to two parts according to an actual position of the leader.

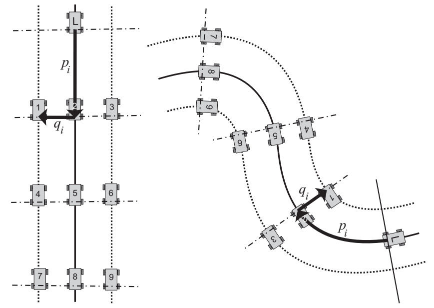
Definition 3. Leader's trajectory Let us define the part of the trajectory that was already travelled by the leader as $\overleftarrow{\Psi}_L(t) := \{\psi(\hat{t}) : t_0 \leq \hat{t} \leq t\}$ and similarly the actual plan that represents the part of the trajectory that should be followed by the leader in future as $\overrightarrow{\Psi}_L(t) := \{\psi(\hat{t}) : t \leq \hat{t} \leq t_f\}$. Then the complete leader's trajectory is simply the union $\Psi(t) := \{\overleftarrow{\Psi}_L(t) \cup \overrightarrow{\Psi}_L(t)\}$.

Many formation control applications require dynamically changing shape of the formation during mission as a respond to changes in environment or in the executed tasks. The approach presented in this article enables utilization of formations with generally varying shape with restrictions given only by kinematic and dynamic limits of the car-like robots. It is evident that the position of a robot within the formation cannot be changed suddenly and such transition must be feasible for the robot.

Less evident, but more important problem for the formation driving of car-like robots is caused by the impossibility to change heading of the robot on the spot. That is why the formations with fixed relative distance in Cartesian coordinates cannot be used. To solve this problem, we utilized an approach in which the followers are maintained in relative distance to the leader in curvelinear coordinates with two axes p and q, where p traces $\Psi(t)$ and q is perpendicular to p as is demonstrated in Fig.1. The positive direction of p is defined from $R_L(\psi_L(t))$ back to the origin of the movement $R_L(\psi_L(t_0))$ and the positive direction of q is defined in the left half plane in direction of forward movement.

The shape of $\mathcal{F}$ is then uniquely determined by states $\psi_L(t_{p_i(t)})$ in *travelled distance* $p_i(t)$ from $R_L(\psi_L(t))$ along $\overleftarrow{\Psi}_L(t)$ and by *offset distance* $q_i(t_{p_i(t)})$ between $\psi_L(t_{p_i(t)})$ and $R_i(\psi_i(t))$ in perpendicular direction from $\overleftarrow{\Psi}_L(t)$. $t_{p_i(t)}$ is time when the leader was at the *travelled distance* $p_i(t)$ behind the actual position. The parameters $p_i(t)$ and $q_i(t)$ defined for each follower i can be vary during the mission.

To convert the state of the followers in curvelinear coordinates to the state in rectangular coordinates, the following equations can be applied:



(a) Constant curvature. (b) Variable curvature.

Fig. 1. Equivalent formations with time invariant Curvelinear coordinates p, q when following different trajectories.

$$\begin{aligned} x_i(t) &= x_L(t_{p_i(t)}) - q_i(t_{p_i(t)})\sin(\theta_L(t_{p_i(t)})) \\ y_i(t) &= y_L(t_{p_i(t)}) + q_i(t_{p_i(t)})\cos(\theta_L(t_{p_i(t)})) \\ \theta_i(t) &= \theta_L(t_{p_i(t)}), \end{aligned} \quad (4)$$

where $\psi_L(t_{p_i(t)}) = (x_L(t_{p_i(t)}), y_L(t_{p_i(t)}), \theta_L(t_{p_i(t)}))$ is the state of the leader in time $t_{p_i(t)}$.

Remark 4. $R_i(\psi_i(t))$, $\forall i \in \{1, ..., n_r\}$ can be determined only if $p_i(t) \geq 0$, $t_0 \leq t \leq t_f$, since $\overleftarrow{\Psi}_L(t)$ that is necessary for defining $R_i(\psi_i(t))$ includes only past information.

VI. Implementation details

The receding horizon framework will be adjusted and implemented for control of the virtual leader as well as of the followers in the following section.

A. Leader trajectory planning and control

In the presented approach we propose to solve two problems: collision free time optimal trajectory planning and the computation of an optimal control sequence, in one optimization step. The aim of the method is to find a control sequence $\mathcal{U}_L(k)$ which could control the leader robot to the target region optimally with respect to a given cost function.

The main idea of the approach is to divide $\mathcal{U}_L(k)$ into two finite time intervals T_N for $k = \{1, ..., N\}$ and T_M for $k = \{N+1, ..., N+M\}$. The first time interval T_N should provide optimal immediate control inputs for $R_L(\psi_L(t))$ regarding local environment of $\mathcal{F}$. Applying this part of planning, the group should be able to respond to changes in workspace such as detecting new obstacles. In this time interval the difference $\Delta t(k+1) = t_{k+1} - t_k$ is a fixed constant (later denoted only Δt). Value of Δt should satisfy the requirements of the classical receding horizon control scheme, because this part of $\mathcal{U}_L(k)$ will be directly used as a control input.

The second interval T_M takes into account information about global characteristic of the environment (complete map) to navigate $\mathcal{F}$ to the goal optimally. The transition points in this part of $\overrightarrow{\Psi}_L(t)$ can be distributed irregularly to effectively cover environment with heterogeneous complexity. During the optimization process, more points should be automatically allocated in the regions with higher density of obstacles or in places where a complicated maneuver of the formation is needed. This is enabled due to the varying values of $\Delta t(k+1) = t_{k+1} - t_k$ for the compact description of the method collected into the vector $\mathcal{T}^{\Delta}_{L,M} = \{\Delta t(N+1), ..., \Delta t(N+M)\}$. The total time of the complete plan from actual position of the robot to the target region will be then $N\Delta t + \sum_{k=N+1}^{N+M} \Delta t(k)$.

To define the trajectory planning problem with two time intervals in a compact form, we need to gather states $\psi_L(k), k \in \{1, ..., N\}$ and $\psi_L(k), k \in \{N+1, ..., N+M\}$ under vectors $\Psi_{L,N} \in \mathbb{R}^{3N}$ and $\Psi_{L,M} \in \mathbb{R}^{3M}$. Similarly the control inputs $\bar{u}_L(k), k \in \{1, ..., N\}$ and $\bar{u}_L(k), k \in \{N+1, ..., N+M\}$ can be gathered under vectors $\mathcal{U}_{L,N} \in \mathbb{R}^{2N}$ and $\mathcal{U}_{L,M} \in \mathbb{R}^{2M}$. All variables describing the complete trajectory from the actual position of the leader until target region are collected as the optimization vector $\Omega_L = [\Psi_{L,N}, \mathcal{U}_{L,N}, \Psi_{L,M}, \mathcal{U}_{L,M}, \mathcal{T}^{\Delta}_{L,M}] \in \mathbb{R}^{5N+6M}$.

1) Objective function and constraints: The trajectory planning and the dynamic obstacle avoidance problem for the virtual leader of $\mathcal{F}$ can be transformed to the minimization of the cost function $J_L(\cdot)$ subject to sets of equality constraints $h_{T_N}(\cdot), h_{T_M}(\cdot)$ and inequality constraints $g_{T_N}(\cdot)$, $g_{T_M}(\cdot)$, $g_{S_F}(\cdot)$, $g_{r_{a,L}}(\cdot)$, that is,

$$\begin{aligned} &\min\ J_L(\Omega_L, \mathcal{O}_{obs}) \\ &\ \text{s.t.}\ h_{T_N}(k-1) = 0, g_{T_N}(k) \leq 0, \forall k \in \{1, ..., N\} \\ &\quad\ h_{T_M}(k-1) = 0, g_{T_M}(k) \leq 0, \forall k \in \{N+1, ..., N+M\} \\ &\quad\ g_{S_F}(\psi_L(N+M)) \leq 0, g_{r_{a,L}}(\Omega_L, \mathcal{O}_{obs}) \leq 0. \end{aligned} \quad (5)$$

The cost function $J_L(\cdot)$ is defined as,

$$\begin{aligned} J_L(\Omega_L, \mathcal{O}_{obs}) := &\sum_{k=N+1}^{N+M} \Delta t(k) \\ &+ \alpha \sum_{j=1}^{n_0} \left(\min\left\{0, \frac{\mathrm{dist}_j(\Omega_L, \mathcal{O}_{obs}) - r_{s,L}}{\mathrm{dist}_j(\Omega_L, \mathcal{O}_{obs}) - r_{a,L}}\right\}\right)^2, \end{aligned} \quad (6)$$

where the objective of the time optimal trajectory planning to reach a desired goal as soon as possible is expressed in the first part of $J_L(\cdot)$ and the influence of the environment on the final solution is added to the cost function in the second term. This second part of $J_L(\cdot)$, that is called an avoidance function, contributes to the final cost when an obstacle from $\mathcal{O}_{obs}$ (static or dynamic) is closer to the trajectory than $r_{s,L}$ and it will approach infinity if a distance $r_{a,L}$ to the obstacle is reached. Distance function $\mathrm{dist}_j(\Omega_L, \mathcal{O}_{obs})$ between obstacle j and $\overrightarrow{\Psi}_L(t)$ then cannot influence optimization if the obstacle is sufficiently far and contrariwise it should exclude unsafe solutions.

The kinematic model (2) with initial conditions given by the actual state of the leader is represented using equality constraints $h_{T_N}(k)$, $\forall k \in \{0, ..., N-1\}$ and $h_{T_M}(k)$, $\forall k \in \{N, ..., N+M-1\}$. The first set of constraints $h_{T_N}(k)$,

$\forall k \in \{0,...,N-1\}$ is applied for the first part of trajectory with constant Δt, while the set $h_{T_M}(k)$, $\forall k \in \{N,...,N+M-1\}$ should be satisfied for the second part where $\Delta t(k)$, $\forall k \in \{N,...,N+M-1\}$ is variable.

Similarly, for the intervals T_N and T_M we define different sets of inequality constraints $g_{T_N}(k)$, $\forall k \in \{1,..,N\}$ and $g_{T_M}(k)$, $\forall k \in \{N+1,...,N+M\}$. In both sets we can find constraints that characterize bounds on the velocity and curvature, as well as inequalities imposing that $\Delta t(k) \geq 0, \forall k \in \{N+1,...,N+M\}$ holds.

The avoidance inequality constraints $g_{r_{a,L}}(\Omega_L, \mathcal{O}_{obs})$ that characterize safety avoidance regions are defined as:

$$r_{a,L}^2 - \text{dist}_j(\Omega_L, \mathcal{O}_{obs})^2 \leq 0, j = \{1,...,n_0\}. \quad (7)$$

Finally $g_{S_F}(\psi_L(N+M))$ is a stability constraint guaranteeing that $\overrightarrow{\Psi}_L(t)$ will enter the target region S_F. For simplification it is supposed that the target region is a circle with radius r_{S_F} centered at C_{S_F}. The stability constraint can be then defined as $r_{S_F} - \|\bar{p}_L(N+M) - C_{S_F}\| \leq 0$.

B. Trajectory tracking for followers

The optimal trajectory $\Psi_L^\circ(t)$ computed for the virtual leader of $\mathcal{F}$, as the result of the previous section, can be directly used for safe navigation of the followers to S_F. Unfortunately, this plan is not able to respond to events behind actual position of the leader. In real application it cannot be assumed that the environment stays static until the last member passes a sector in which previously collision free plan was created by the leader moving ahead of the group. We also cannot expect that each follower will follow its optimal plan faultlessly. Incorrect driving direction or velocity can be dangerous for neighbors, that should be able to avoid possible collisions.

The idea of our approach is to use leader trajectory that consists of $\overleftarrow{\Psi}_L(t)$ and the part of $\overrightarrow{\Psi}_L^\circ(t)$ for $t \leq N\Delta t$ that reflects local environment, as an input $\psi_L(t_{p_i(t)})$ for the formation driving approach described in Section V. The states obtained by applying equations (4) for $t = t_0, t_0+\Delta t, ..., t_0+(N-1)\Delta t$ then can be utilized as desired states $\psi_{d,i}(k) = (\bar{p}_{d,i}(k), \theta_{d,i}(k))$, $k = \{1,...,N\}$, $i = \{1,...,n_r\}$ for trajectory tracking algorithm with an obstacle avoidance function.

Similarly to the leader planning in Section (VI-A) for each follower i the state and the control vectors $\psi_i(k), k = \{1,...,N\}$ and $\bar{u}_i(k), k = \{1,...,N\}$ can be gathered as vectors $\Psi_i \in \mathbb{R}^{3N}$ and $\mathcal{U}_i \in \mathbb{R}^{2N}$ and then collected to an unique optimization vector $\Omega_i = [\Psi_i, \mathcal{U}_i] \in \mathbb{R}^{5N}$. The vector Ω_i will be used to represent the dynamic behavior of the discrete trajectory tracking with collision avoidance and to capture it as a static optimization process under the receding horizon scheme described in Section IV.

1) Objective function and constraints: The discrete trajectory tracking for $R_i(\psi_i(t))$, $i = \{1,...,n_r\}$ is in this section transformed to an optimization problem with cost function $J_i(\cdot)$ subject to a number of equality constraints $h_i(\cdot)$ and inequality constraints $g_i(\cdot)$, $g_{r_a}(\cdot)$, $g_{ra,i}(\cdot)$. The optimization process for each follower is decentralized and could be computed on board of the follower vehicles. The only necessary communication within the group is used for propagation of the appropriate part of the leader trajectory and for sharing the positions of the obstacles detected by the other members of the team. The complete optimization scheme can be presented in the form:

$$\begin{aligned} &\min\ J_i(\Omega_i, \mathcal{O}_{obs}), i = \{1,...,n_r\} \\ &\text{s.t. } h_i(k-1) = 0, g_i(k) \leq 0, \forall k \in \{1,...,N\} \\ &\quad g_{r_a}(\Omega_i, \mathcal{O}_{obs}) \leq 0, g_{r_{a,i}}(\Omega_i, \bar{p}_{last}) \leq 0 \end{aligned} \quad (8)$$

where

$$\begin{aligned} J_i(\Omega_i, \mathcal{O}_{obs}) := &\sum_{k=1}^{N} \|(\bar{p}_{d,i}(k) - \bar{p}_i(k))\|^2 \\ &+ \alpha \sum_{j=1}^{n_0} \left(\min\left\{0, \frac{\text{dist}_j(\Omega_i, \mathcal{O}_{obs}) - r_s}{\text{dist}_j(\Omega_i, \mathcal{O}_{obs}) - r_a}\right\}\right)^2 \\ &+ \beta \sum_{j \in \bar{n}_n} \left(\min\left\{0, \frac{d_{i,j}(\Omega_i, \Omega_j^\circ) - r_{s,i}}{d_{i,j}(\Omega_i, \Omega_j^\circ) - r_{a,i}}\right\}\right)^2 . \end{aligned} \quad (9)$$

The proposed cost function $J_i(\cdot)$ consists of three components with their influence adjusted by constants α and β. The first component represents deviations of the position $\bar{p}_i(k)$, $\forall k \in \{1,...,N\}$ from the desired position $\bar{p}_{d,i}(k)$, $\forall k \in \{1,...,N\}$ and so it is crucial for the effort of the trajectory tracking. The desired position $\bar{p}_{d,i}(k)$, $\forall k \in \{1,...,N\}$ is derived from the output of the leader's plan using the set of equations in (4). The second summation term in $J_i(\cdot)$ is equivalent to the second term used in (6) and should ensure that dynamic or lately detected obstacles will be avoided. Only difference is that the avoidance regions for single robots can be smaller than the avoidance regions for the leader as was explained in Section V. The third component of the equation (9) is the avoidance function in which the other members of the team are also considered as dynamic obstacles. This part has to protect the robots in cases of an unexpected behavior of a defective neighbors. Function $d_{i,j}(\Omega_i, \Omega_j^\circ)$ is the minimal distance between the planned trajectory of $R_i(\psi_i(t))$ and the actually exercised plan of $R_j(\psi_j(t))$, $j \in \bar{n}_n$, where $\bar{n}_n = \mathcal{F} \backslash i$.

The equality constraints $h_i(k)$, $\forall k \in \{0,...,N-1\}$ defined in (8) are identical to the equality constraints $h_{T_N}(k)$, $\forall k \in \{0,...,N-1\}$ used in (5). Similarly the inequality constraints $g_i(k)$, $\forall k \in \{1,...,N\}$ in (8) are identical to the inequality constraints $g_{T_N}(k)$, $\forall k \in \{1,...,N\}$ used in (5), and finally the avoidance inequality constraints $g_{r_a}(\Omega_i, \mathcal{O}_{obs})$ and $g_{r_{a,i}}(\Omega_i, \bar{p}_{last})$ are given as $r_a^2 - \text{dist}_j(\Omega_i, \mathcal{O}_{obs})^2 \leq 0, j = \{1,...,n_0\}$ and $r_{a,i}^2 - d_{i,j}(\Omega_i, \bar{p}_{last,j})^2 \leq 0, j \in \bar{n}_n$, similarly as $g_{r_{a,L}}(\Omega_L, \mathcal{O}_{obs})$ in equation (7).

VII. Results

In this section we provide a numerical example of the proposed method in a dynamically changing environment.

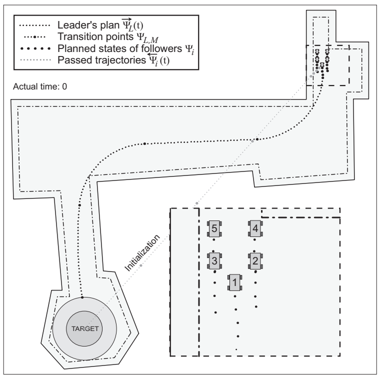


Fig. 2. Initial Configuration

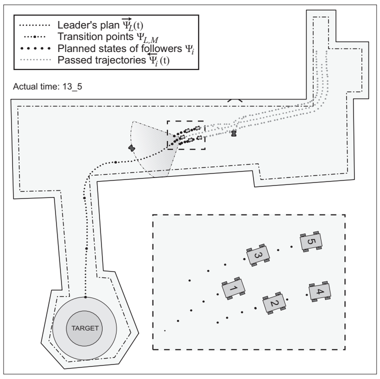


Fig. 3. Intermediate plan performance

As the workspace of the robots, a map of the computer science building at the University of Würzburg, Germany, was chosen. This map, depicted in Fig.2, is known by the robots at the beginning of the mission, whereas inner obstacles (static as well as dynamic) are unknown and will be detected during the task accomplishment. In the experiment four followers (robots 2-5) leaded by one leader (robot 1) were used. Nevertheless due to the architecture of the trajectory planning, robot 1 is considered as a follower with distance $q_1 = 0$ and $p_1 = 0$ from the virtual leader. The input constants of the algorithm defined in Section VI-A were adjusted as $N = 4$, $M = 4$, $n = 2$, $\Delta t = 0.25$. In Figures 3 and 4 it can be observed how the formation successfully avoids dynamic and static obstacles (previously unknown) until reaching the desired target area.

VIII. Conclusions

In this paper we have presented a RHC approach to formation driving of car-like robots reaching a target area in minimal time. The proposed approach is novel in the sense that takes advantage of the parametrization of control inputs, enabling to perform reactive near future control as well as further path planning in time. Successful numerical results were reported to illustrate the effectiveness of the proposed approach.

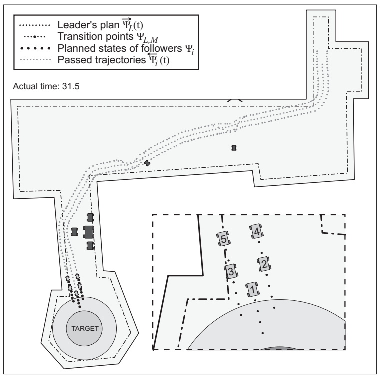


Fig. 4. Final goal reached

IX. Acknowledgments

This work was supported in part by the Elite Network of Bavaria through the program Identification, Optimization and Control with Applications in Modern Technologies. This work was also supported in part by the Boeing Company via the Information Trust Institute, University of Illinois at Urbana Champaign, and by the Alexander von Humboldt Foundation, Germany.

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